Superconducting critical temperature under pressure

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ABSTRACT

The present record on the critical temperature of a superconductor is held by sulfur hydride (approx. 200 K) under very high pressure (approx. 56 GPa). As a consequence, the dependence of the superconducting critical temperature on pressure became a subject of great interest and a high number of papers on different aspects of this subject have been published in the scientific literature since. In this paper, we calculate the superconducting critical temperature as a function of pressure, $T_c(P)$, by a simple method. Our method is based on the functional derivative of the critical temperature with the Eliashberg function, $\delta T_c(P)/\delta \alpha^2 F(\omega)$. We obtain the needed coulomb electron-electron repulsion parameter, $\mu^*(P)$ at each pressure in a consistent way by fitting it to the corresponding $T_c$ using the linearized Migdal–Eliashberg equation. This method requires as input the knowledge of $T_c$ at the starting pressure only. It applies to superconductors for which the Migdal–Eliashberg equations hold. We study Al and $\beta$–Sn two weak-coupling low-$T_c$ superconductors and Nb, the strong coupling element with the highest critical temperature. For Al, our results for $T_c(P)$ show an excellent agreement with the calculations of Profeta et al., which are known to agree well with experiment. For $\beta$–Sn and Nb, we found a good agreement with the experimental measurements reported in several works. This method has also been applied successfully to PdH elsewhere. Our method is simple, computationally light and gives very accurate results.

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1. Introduction

One of the goals of the research in superconductivity nowadays is to find a system with a room critical temperature. Due to the success with the sulfur hydride under pressure [1] the detailed understanding of the different properties of a material that contribute to enhance its critical temperature, became of central interest. For this reason, one of the present lines of research is the dependence of the critical temperature on pressure, $T_c(P)$. In electron-phonon superconductors, pressure affects the vibration spectrum by shifting it to higher frequencies which can enhance or lower the critical temperature depending on details of the system under study. Also the electron-phonon interaction is affected by pressure. This is put in evidence by several experimental works [2–10].

In order to contribute to the understanding of the details that determine whether or not pressure will enhance $T_c$, we have developed a simple but still quite accurate method to calculate the superconducting critical temperature as a function of pressure, $T_c(P)$. We use the density functional theory (DFT) and the density functional perturbation theory [11–13] (DFPT) to get the electron and phonon band structures and the Eliashberg function, $\alpha^2 F(\omega)$, from first principles. We use the Quantum Espresso suite codes [14,15] for most of the calculations made in this work. This method applies to superconductors for which the Migdal–Eliashberg equations (MEE) [16,17] are valid to describe their superconducting properties as it is the case of the electron-phonon superconductors. The parameters that enter the linearized MEE (LMEE) are the critical temperature $T_c$, which can be obtained from resistivity experiments, for example, the frequency at which the sum over the Matsubara frequencies on the frequency imaginary axis is stopped, the so-called, cut-off frequency, $\omega_c$, which can actually be fixed numerically, and the electron-electron repulsion parameter, $\mu^*$. The electron-phonon interaction parameter, $\lambda$, is known once the Eliashberg function, $\alpha^2 F(\omega)$, is known and it can also be obtained from specific heat experiments, for example. The electron-electron repulsion parameter, $\mu^*$, requires some attention. Actually, it has not been yet neither calculated nor measured with enough precision to be useful as the parameter needed to solve the LMEE to obtain an accurate value for $T_c$. If $T_c$ is known, then by solving the LMEE we can fix $\mu^*$. When $T_c$ is not known, several ways have been proposed to estimate $\mu^*$ so that it can be calculated from the LMEE. PMorel and Anderson [18] suggest the following analytic formula

$$\mu^* = \frac{\mu}{1 + \mu \ln(\frac{\omega_c}{\mu})}$$

(1)
where the dimensionless parameter $\mu = -V > N(E_F)$ is the product of the averaged screened coulomb interaction, $V$, and the density of states at the Fermi energy, $N(E_F)$. $E_d$ and $\omega_{ph}$ are the electron and phonon energy scales, respectively. Further, Bennemann and Garland [19], Smith [4] and Neve et al. [20] give semi-empirical formulas to estimate the behavior of the coulomb pseudo-potential as a function of pressure. Liu et al. [21] and Freericks et al. [22] calculated $\mu^*$ scaled to the maximum phonon frequency, meaning to replace $\omega_{ph}$ in Eq. (1) by $\omega_{max}$, the maximum phonon frequency. Daams and Carbote [23] fit $\mu^*$ solving the LME using the experimental value of $T_c$. In a more recent work Bauer et al. [24] calculated corrections to $\mu^*$ based on the Hubbard–Holstein model. There is no consensus concerning the proper way to calculate $\mu^*$ under pressure or even at ambient pressure. For example, for Nb at ambient pressure, a set of different values for $\mu^*$ are reported, 0.117 [25], 0.13 [20], 0.14 [26], 0.183 [27], 0.21 [28] and 0.249 [29] which differ considerably from each other.

The rest of the paper is organized as follows. In Section 2, we present the theoretical foundations that support our method. The method is described in Section 3. In the next Section 4, we describe some technical details of the calculation. In Section 5, we present our results and compare them with other work and to experiment. For Al, with the known successful calculations of Profeta et al. [9] and with experiment [2,3,8]; for Sn, with the experimental measurements of Smith and Chu [30], Berman et al. [5] and Jennings and Swenson [6]. We get a good agreement with them. For Nb, we compare our results to the experimental ones of Struzhkin et al. [7]. Finally, we summarize our work in a final Section 6.

2. Theoretical foundations

2.1. The functional derivative

The functional derivative, $\frac{\delta E_F}{\delta \alpha F(\omega)}$, can be thought as a measure of the strength of the influence of a particular frequency on $T_c$. In this sense, a calculation of the functional derivative tells us how favorable a certain frequency range is for an increase in the transition temperature. The maximum of the functional derivative, $\omega_{opt}$, is therefore the most important region in determining the critical temperature. Actually very useful relation between this optimum frequency and $T_c$ does exist, namely $\hbar \omega_{opt} = C_{K} T_c$ where $C$ is a constant between 7–8 and $K_0$ is the Boltzmann constant.

From the solution of the LME and using the algorithm of Leavens [31] the functional derivative can be obtained. Changes in $T_c$, calculated using this method have been considered previously by several authors to study the influence on $T_c$ due to changes in concentration [32] and changes in composition [33]. Also for a two-band superconductor as MgB$_2$ [34] or, as in our case, to study the influence of pressure [23,35].

2.2. Linearized Migdal–Eliashberg equations

Central to our calculations is the functional derivative of $T_c$, with respect to the Eliashberg function, $\alpha^2 F(\omega, P_i)$, which we obtain from the solution of the Linearized Migdal–Eliashberg equations (LME) [16,17]. As stated before, we solve LME to fit $\mu^*$ to the corresponding value of $T_c$. On the imaginary axis, the LME is:

$$\rho \tilde{\Delta}_n = \pi T \sum_m \left( \lambda_{mn} - \mu^* \right) - \delta m \frac{|\omega_n|}{\pi T} \tilde{\Delta}_m,$$

$$\tilde{\Delta}_n = \frac{\tilde{\Delta}_n}{\rho + |\omega_n|} \Delta_m,$$

$$\lambda_{mn} = 2 \int_0^\infty \frac{d\omega \alpha^2 F(\omega)}{\omega^2 + (\omega_n - \omega_m)^2},$$

(5)

(6)

(7)

where $\tilde{\Delta}$ is the gap function, $\omega_n$ is the Matsubara frequency, $\rho$ is the pair breaking parameter and $n = 0, \pm 1, \pm 2, \ldots$. In particular, $\lambda_{mn} = \lambda$ is the electron-phonon coupling constant. We take $\omega_i$ to be 10 times the maximum phonon frequency, $\omega_{max}$, as suggested by Bergmann and Rainer [56]. The Eliashberg function is defined as $\alpha^2 F(\omega) = \frac{1}{N(\epsilon_F)} \sum_{m} \sum_n \delta (\omega - \omega_{nm}) \times \sum_{n} [g_{k,q,m}^{n,m}]^2 \delta (\epsilon_{\tilde{q},q} - \epsilon_{n}) \tilde{\Delta} (\epsilon_{\tilde{q},q} - \epsilon_{n}) - \Delta (\epsilon_{\tilde{q},q} - \epsilon_{n})$.

3. The method

To calculate the critical temperature as a function of pressure, we need only as the starting data, the critical temperature at the starting pressure, $T_{c0}$. We make use of the espresso codes [13] to optimize the lattice constants first and then to calculate the Eliashberg function, $\alpha^2 F(\omega, P_i)$, from first principles at the starting pressure. To get the electron-electron repulsion parameter, $\mu^*$, we solve the Linearized Migdal Eliashberg Equation (LME) to fit it to the known $T_{c0}$. We use the Mc Master programs for that purpose [23,25,36,37,55]. With these data, we calculate the functional derivative of $T_c$ with the Eliashberg function, $\alpha^2 F(\omega)$ at $P_i$, $\delta E(F(P_i))/\delta \alpha^2 F(\omega)$ [19]. Now, we define the next pressure, say $P_{i+1}$, obtain the new lattice parameters, optimize them and get the new Eliashberg function, $\alpha^2 F(\omega, P_{i+1})$. This new Eliashberg function differs from the previous one by

$$\Delta \alpha^2 F(\omega)_{h_{i+1}} = \Delta \alpha^2 F(\omega, P_{i+1}) - \Delta \alpha^2 F(\omega, P_i)$$

(8)

The change in the critical temperature is now obtained from (Rainer and Bergman [38], and Baquero and Lopez-Olazagasti [39])

$$\Delta T_{c_{i+1}} = \int_0^\infty \frac{\delta E(F(P_i))}{\delta \alpha^2 F(\omega)} \Delta \alpha^2 F(\omega)_{h_{i+1}} d\omega$$

(9)

and then, the $T_{c_{i+1}}$, at the new pressure is calculated as

$$T_{c_{i+1}} = T_{c0} + \Delta T_{c_{i+1}}.$$

(10)

4. Technical details

We, first, relax the internal degrees of freedom and the lattice vectors of the corresponding structure (Al, $\beta$-Sn and Nb) using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) quasi-Newton algorithm at each pressure to get the corresponding lattice constants. From these relaxed structure configurations, we calculated the electronic and phonon band structures, electron (DOS) and phonon (PHDOS) densities of states, and the Eliashberg function $\alpha^2 F(\omega)$. We used a kinetic energy cut-off of 60 Ry for the expansion of the wave function into plane waves and 240 Ry for the density. To integrate over the Brillouin zone (BZ), we used for the electronic integration a k-grid of $32 \times 32 \times 32$ and for the phononic integration a q-grid of $8 \times 8 \times 8$ according to the Monkhorst-Pack
Table 1

Superconducting properties of Al under pressure. The experimental data of $T_c$ were obtained by linear interpolation from Profeta et al. [9], Gusber and Webb [3] and Levy and Olsen [2].

<table>
<thead>
<tr>
<th>P[GPa]</th>
<th>a[Bohr]</th>
<th>$\lambda$</th>
<th>$\mu^*$</th>
<th>$T^{exp}_c[K]$</th>
<th>$T^{calc}_c[K] \pm [2,3]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>7.5460</td>
<td>0.4259</td>
<td>0.1415</td>
<td>0.460156</td>
<td>1.18</td>
</tr>
<tr>
<td>0.5</td>
<td>7.6297</td>
<td>0.4168</td>
<td>0.1408</td>
<td>0.722382</td>
<td>1.078</td>
</tr>
<tr>
<td>1.0</td>
<td>7.6141</td>
<td>0.4084</td>
<td>0.1402</td>
<td>0.404516</td>
<td>0.969</td>
</tr>
<tr>
<td>1.5</td>
<td>7.5989</td>
<td>0.4009</td>
<td>0.1398</td>
<td>0.70122</td>
<td>0.872</td>
</tr>
<tr>
<td>2.0</td>
<td>7.5841</td>
<td>0.3942</td>
<td>0.1396</td>
<td>0.67337</td>
<td>0.791</td>
</tr>
<tr>
<td>2.5</td>
<td>7.5695</td>
<td>0.3870</td>
<td>0.1391</td>
<td>0.47413</td>
<td>0.706</td>
</tr>
<tr>
<td>3.0</td>
<td>7.5553</td>
<td>0.3805</td>
<td>0.1388</td>
<td>0.18422</td>
<td>0.634</td>
</tr>
<tr>
<td>3.5</td>
<td>7.5415</td>
<td>0.3743</td>
<td>0.1387</td>
<td>0.44337</td>
<td>0.567</td>
</tr>
<tr>
<td>4.0</td>
<td>7.5280</td>
<td>0.3680</td>
<td>0.1385</td>
<td>0.37383</td>
<td>0.505</td>
</tr>
<tr>
<td>4.5</td>
<td>7.5148</td>
<td>0.3623</td>
<td>0.1387</td>
<td>0.36808</td>
<td>0.448</td>
</tr>
<tr>
<td>5.0</td>
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<td>0.3569</td>
<td>0.1393</td>
<td>0.30872</td>
<td>0.392</td>
</tr>
<tr>
<td>5.5</td>
<td>7.4892</td>
<td>0.3518</td>
<td>0.1410</td>
<td>0.77993</td>
<td>0.324</td>
</tr>
<tr>
<td>6.0</td>
<td>7.4766</td>
<td>0.3460</td>
<td>0.1520</td>
<td>0.44223</td>
<td>0.204</td>
</tr>
</tbody>
</table>

scheme [40]. We performed the calculations using the pseudopotential-plane wave (PW) method of Perdew et al. [41,42], the generalized gradient approximation (GGA) and a Troullier and Martins [43] norm-conserving pseudopotential. The cut-off and grids were chosen big enough to obtain a good precision in $\alpha^2 F(\omega)$ and $\lambda$ calculated within the density-functional perturbation theory (DFPT) frame [11,12]. We used the Quantum Espresso code [13,14] for all these calculations.

5. Results and discussion

5.1. Aluminium

We first apply the method to the weak-coupling superconductor fcc Aluminium. Its $T_c = 1.8$K ([27]) at ambient pressure which we take as our starting pressure in this case and follow the steps of the method just described. Our results are presented in the Table 1 and in Fig. 1. Our results are in excellent agreement with the work of Profeta et al. [9].

In Fig. 1, we can see that we get the trend of $T_c(P)$ right. We stress that our calculation coincides exactly with the one of Profeta et al. [9]. It is, nevertheless, interesting that both theoretical calculations by different methods coincide exactly with each other but not as exactly with experiment. The difference might be due to details of the sample not included in the theoretical calculations.

In Table 1, in the first column we report the pressure in GPa at which the calculation is done; in the second the lattice parameter in Bohr, in the third, the corresponding electron-phonon interaction parameter, $\lambda$; in the fourth, the electron-electron repulsion parameter, $\mu^*$; in the fifth column we present our calculated $T^{calc}_c$ in Kelvin and in the last column sixth, we present the corresponding experimental value (linear interpolation from Gusber and Webb [3] and Levy and Olsen [2] data).

As we can see from Table 1, as pressure increases from 0.0 GPa to 6.0 GPa, the corresponding critical temperature decreases steadily from 1.18 K to 0.204 K. Further, in Fig. 1, we can also see that in spite of the excellent agreement with the SCDF results, our method gives results closer to experiment in the 5.5 to 6.0 GPa interval. The lattice parameter, $a$, decreases from 7.646 Bohr to 7.4766 Bohr and the electron-phonon interaction parameter, $\lambda$, decreases steadily from 0.4259 to 0.346. This last behavior is expected and correlates with the behavior of $T_c$. Notice that the electron-electron repulsion parameter, $\mu^*$, presents a mild minimum at 4.0 GPa which does not correlate with any other parameter anyhow.

We conclude that our calculation for Al is in excellent agreement with the one of Profeta et al. [9] which compare well with experiment although our method seems to give results nearer to the experimental ones in the 5.5–6 GPa range (see Fig. 1). We also present a detailed Table 1 where the evolution with pressure of several parameters is reported.

5.2. Niobium

In this work, we calculate the critical temperature, $T_c(P)$, by the functional density method, FDM (our method), for the whole interval between 0–6.5 GPa including the interval from 2.41–3.9 GPa where no experimental data exist. We establish that a minimum of the critical temperature as a function of pressure does indeed exists for Nb under pressure at 3.9 GPa where the experimental $T_c = 9.1$ K. Corresponding data are shown in Table 2 and Fig. 2.

Nb is the superconductor with the highest critical temperature among the elements at ambient pressure ($T_c = 9.25$K [46]). Static compression of Nb up to 54 GPa and shock-wave experiments up to
170 GPa show no evidence for phase transitions so that the changes in $T_c$ are expected to be related not to structural changes but to changes in the electronic properties [7]. Early experiments by Smith [4] on Nb from 0 up to 2.4 GPa found $T_c$ to decrease mono-

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
P[GPa] & $\rho$[bohr] & $\lambda$ & $\mu^*$ & $T_c^{\text{FDM}}$ & $\rho^{\text{Nb}}$ & $\rho^{\text{FDM}}$ \\
\hline
0 & 6.2973 & 1.2106 & 0.297251 & 9.25 & 9.21 & 9.21 \\
0.5 & 6.2915 & 1.2106 & 0.296646 & 9.24 & 9.21 & 9.26 \\
1 & 6.2857 & 1.2080 & 0.296249 & 9.23 & 9.20 & 9.31 \\
1.5 & 6.2799 & 1.2050 & 0.296109 & 9.15 & 9.19 & 9.36 \\
2.4 & 6.2697 & 1.1905 & 0.295946 & 9.10 & 9.18 & 9.46 \\
3 & 6.2631 & 1.1924 & 0.295604 & 9.09 & 9.52 & 9.52 \\
3.5 & 6.2576 & 1.1914 & 0.295581 & 9.07 & 9.55 & 9.55 \\
3.9 & 6.2532 & 1.1910 & 0.295725 & 9.05 & 9.57 & 9.57 \\
4.5 & 6.2467 & 1.1921 & 0.295620 & 9.08 & 9.60 & 9.60 \\
5 & 6.2414 & 1.1932 & 0.295776 & 9.10 & 9.63 & 9.63 \\
5.5 & 6.2360 & 1.1958 & 0.295776 & 9.15 & 9.74 & 9.65 \\
6 & 6.2308 & 1.1987 & 0.295613 & 9.20 & 9.75 & 9.68 \\
6.5 & 6.2256 & 1.1998 & 0.295761 & 9.23 & 9.76 & 9.58 \\
7 & 6.2204 & 1.2047 & 0.295753 & 9.29 & 9.77 & 9.57 \\
7.5 & 6.2153 & 1.2048 & 0.295534 & 9.33 & 9.78 & 9.57 \\
8 & 6.2103 & 1.2030 & 0.295773 & 9.37 & 9.78 & 9.57 \\
8.5 & 6.2053 & 1.2069 & 0.295757 & 9.43 & 9.83 & 9.57 \\
9 & 6.2004 & 1.2093 & 0.295853 & 9.52 & 9.87 & 9.57 \\
10 & 6.1907 & 1.2146 & 0.296511 & 9.68 & 9.97 & 9.57 \\
\hline
\end{tabular}
\caption{Superconducting properties of Nb under pressure. The experimental critical temperature of Nb, $T^{\text{FDM}}_c$ (measurement of $T_c$ with medium by compression) and $T^{\text{FDM}}_c$ (measurement of $T_c$ with medium by decompression) are obtained by linear interpolation of experimental measurement of Struzhkin et al. [7] and Smith [4]. $T_c^{\text{SM}}$ is the $T_c$ calculated in the present work by FDM.}
\end{table}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{(color on line) The experimental critical temperature reported by Smith [4], and Struzhkin et al. [7]: M. Comp. (measurement with medium by compression) and M. Dec. (measurement with medium by decompression). The FDM curve is the $T_c$ calculated in the present work by FDM.}
\end{figure}

\textsuperscript{a} $\lambda$ obtained from the electronic tunneling spectroscopy is 1.22 [44.45].
\textsuperscript{b} The start point from FDM to low pressure interval taken of experimental measurement [46].

The interval 0 to 1.0 GPa we found that $T_c$ decreases monotonically with pressure at a rate of $\frac{\partial T_c}{\partial P} = -0.02 \, \text{K/GPa}$ in full agreement with the findings of Smith [4]. In the interval 1 to 2.4 GPa, we get a $T_c$ dependence on pressure steeper, $-0.06 \, \text{K/GPa}$. In general, the behavior of $T_c$ under pressure is not linear. We obtained the minimum $T_c$ at 9.05 K at 3.9 GPa in good agreement with the value of $T_c$ obtained by Struzhkin et al. [7] of 9.11 K (Table 2 and Fig. 2).

Now we analyze the character of the minimum $T_c$ at 3.9 GPa. In Table 2, we summarize our results. The lattice constant, $a$, steadily decreases under pressure non-linearly. $T_c$ decreases up to 3.9 GPa to about 9–10 GPa and then the $T_c$ decreases slowly up to about 30 GPa and becomes more or less constant up to about 70 GPa in the case of hydrostatic compression in a NaCl medium. In no-medium compression the $T_c$ dependence on pressure is different. It rises slowly up to about 30 GPa and then steadily decreases up to 130 GPa. Struzhkin et al. [7] find anomalies (unexpected changes in the trend) in $T_c$ at 5–6 GPa where it increases by about 0.7 K and at 50–60 GPa where it decreases by about 1 K. These anomalies are punctual events that require specific explanations valid only at the pressure where they occur. Tse et al. [48] investigated theoretically the Hall coefficient under pressure using full potential linearized augmented plane-waves (FPLAPW) and the electron-phonon coupling parameter $\lambda$ using full potential linear muffin-tin orbitals (FPLMTO) and pseudo potential plane-waves (SPPW). This study confirms the occurrence of two electronic topological transitions at 5 and 60 GPa. The Fermi surface changes but very little. The small changes in the Fermi surface have a subtle effect on the transport coefficients which manifest themselves as discontinuities in the electronic transport properties of the metal and also discard non-hydrostatic uniaxial stress as responsible for the behavior of $T_c$ at 50–60 GPa [7], Wierzbowska et al. [49] use the nesting factor, in order to detect tiny features on the Fermi surface; the nesting factor is high in all directions of the Brillouin zone (BZ) in the vicinity of the $\Gamma$ point where the nearby phonon frequencies are small and their linewidths are nonvanishing enabling a very large contribution to the electron-phonon coupling in the Nb case. The exact relation between pressure and $T_c$ has been studied through a detailed analysis of the electronic bands [29,48,50,51], the Fermi surface [49,52] and the electron-phonon coupling constant [48]. Further, between 70 and 132 GPa (no-medium) $T_c$ drops continuously to 4.7 K.
where a minimum of $T_c$ occurs and increases back steadily. This minimum is confirmed experimentally and is due to a local decrease in the electron-phonon interaction parameter, $\lambda$, as it can be seen in the third column of Table 2. This minimum is coincident with a slightly higher value of the electron-electron interaction parameter $\mu^*$ (fourth column in the Table 2).

If we proceed to the next interval where $T_c$ increases very rapidly with pressure, we get an increase but much slower that the experiment (Fig. 2). Since we are facing actually two different experiments, i.e., one from $P = 0$ GPa to $P = 3.9$ GPa and another from $P = 3.9$ GPa.

5.3. Tin

We report here our results for $T_c(P)$ for $\beta$ – Sn. This structure is stable in the pressure interval 0–100 GPa [53]. We have used the same method as described above in this paper. Our results (0–8 GPa) are presented in the Table 3 and in the Fig. 3. We compare our results with the experimental measurements of Smith [4], Berman et al. [5], and Jennings and Swenson [6]. $T_c(P)$ decreases with pressure in the whole interval under consideration. This might be due to the decreasing behavior of $\lambda$ with increasing pressure, a behavior that correlates with the one of electron-electron interaction parameter $\mu^*$ (see Table 3).

6. Conclusions

In conclusion, we have studied the behavior of $T_c$ of three elements (Al, $\beta$-Sn and Nb) under pressure using the Quantum Espresso codes suite [14,15] and the Mc Master set of programs [23,25,36,37,55]. Our results are in a quite good agreement with the experimental data. Both Al and $\beta$-Sn show a decreasing $T_c$ as pressure increases. It correlates with the weakening of electron-phonon interaction parameter $\lambda$ and with a slow increasing of the electron-electron repulsion parameter $\mu^*$ as pressure increases. This is in contrast to the behavior of Nb where the $T_c$ diminishes from 0 to 3.9 GPa where it shows a minimum (at 3.9 GPa). This behavior correlates with a parallel decrease of the electron-phonon interaction parameter $\lambda$ together with an increase of the electron-electron interaction parameter $\mu^*$ (see Table 2). Our method is simple but still quite accurate and represents a mean to explore the different properties of a system that contribute to enhance the critical temperature of a superconductor when pressure is applied.

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